Definitions

- **Network** – Any structure containing interconnected elements.
- **Circuit** – Usually physical structure constructed from electrical components.

(A) **Linear Network**: response proportional to excitation. Superposition applies:

$$If \quad e_1(t) \rightarrow w_1(t) \quad and \quad e_2(t) \rightarrow w_2(t)$$

Then

$$k_1 \cdot e_1(t) + k_2 \cdot e_2(t) \rightarrow k_1 \cdot w_1(t) + k_2 \cdot w_2(t)$$

ideal only

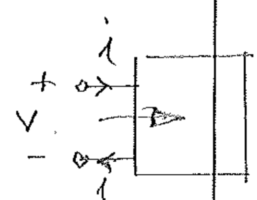
(B) **Time-Invariant Network**: $e(t) \rightarrow w(t)$ relation the same if $t \rightarrow t + t_1$. Time varying otherwise.

(C) **Passive Network**: EM energy delivered always non-negative. Specifically:

$$E(t) = \int_{-\infty}^{t} \underbrace{v(x)i(x)}_{P}\,dx \geq 0$$

or

associated directions



signal energy

$$E(t) = \int_{t_0}^{t} v(x)i(x)dx + E(t_0) \geq 0$$

reference directions

This must be true for any voltage and its resulting current for all t.

Otherwise, active.

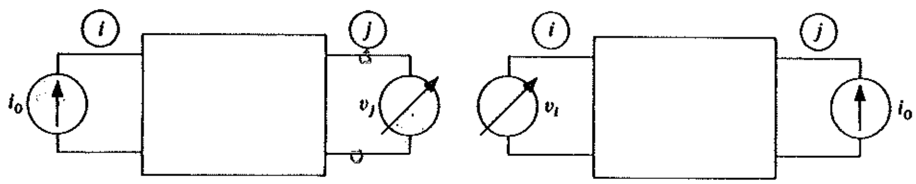
(D) **Lossless Circuit**: input energy is always equal to the energy stored in the network. Otherwise, lossy.

(E) **Distributed Network**: must use Maxwell's equation to analyze. Examples: transmission lines, high speed VLSI circuits, PCBs.

Lumped

(F) **Memoryless or Resistivity Circuit**: no energy storing elements. Response depends only on instantaneous excitation. Otherwise, dynamic or memoried circuit.

(G) **Reciprocity**: response remains the same if excitation and response locations are interchanged. Specifically:

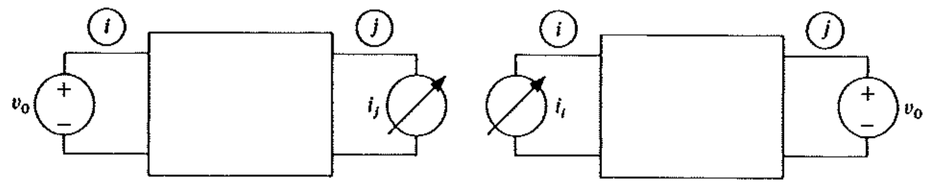


$Z_{12} = Z_{21}$

$R_{in} \to \infty$

$v_i = v_j$

(a)

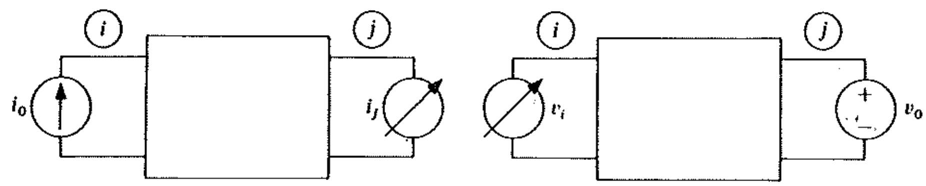


$y_{12} = y_{21}$

$R_{in} = 0$

$i_j = i_i$

(b)



∞ $\qquad$ 0 $\qquad$ $\frac{i_j}{i_0} = \frac{v_i}{v_0}$

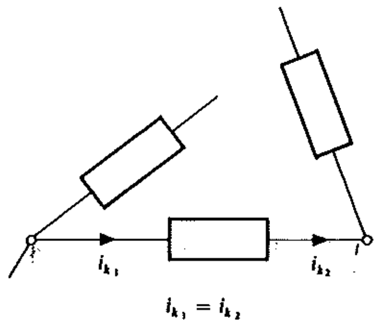
(c)

Otherwise, non-reciprocal.

$h_{12} = -h_{21}$

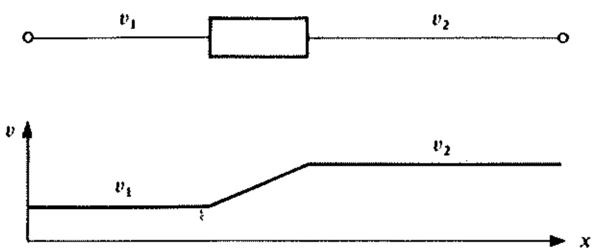
$g_{12} = -g_{21}$

(H) **Lumped Network**: physical dimensions can be considered zero. In reality, much smaller than the wavelength of the signal.

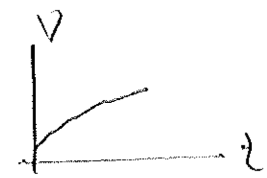


(*a*)

KCL
KVL
Branch eqn.



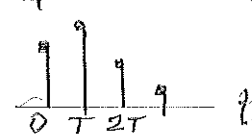
(*b*)



(I) **Continuous-Time Circuit**: the signals can take on any value at any time.

Discrete-Time

(J) **Sampled-Data Circuit**: the signals have a known value only at some discrete time instances. Digital, analog circuits.



An ideal RLC circuit is linear, time-invariant, passive, lossy, reciprocal, lumped, dynamic continuous-time network.

(A) **Ideal R, L, C:** ≥ 0

Element	Parameter	Voltage-Current Relationship: Direct	Voltage-Current Relationship: Inverse	Symbol
Resistor	Resistance R Conductance G	$v = Ri$	$i = \frac{1}{R}v = Gv$	i, R, $+\,v\,-$
Inductor	Inductance L Inverse Inductance Γ	$v = L\frac{di}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(x)dx + i(0)$	i, L, $+\,v\,-$
Capacitor	Capacitance C Elastance D	$i = C\frac{dv}{dt}$	$v(t) = \frac{1}{C}\int_0^t i(x)dx + v(0)$	i, C, $+\,v\,-$

duals

Associated directions for $\underline{v}$ & $\underline{i}$

Table 1

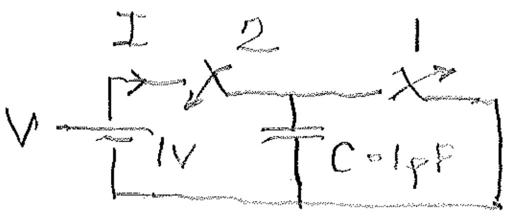
Each passive;

Assuming standard references, the energy delivered to each of the elements starting at a time when the current and voltage were zero will be:

$$E_R(t) = \int_{-\infty}^{t} Ri^2(x)dx \geq 0 \qquad R > 0 \tag{67}$$

$$E_L(t) = \int_{-\infty}^{t} L\frac{di(x)}{\cancel{dx}}i(x)\cancel{dx} = \int_{0}^{i(t)} Li'di' = \frac{1}{2}Li^2(t) \geq 0 \tag{68}$$

$$E_C(t) = \int_{-\infty}^{t} C\frac{dv(x)}{dx}v(x)dx = \int_{0}^{v(t)} Cv'dv' = \frac{1}{2}Cv^2(t) \geq 0 \tag{69}$$



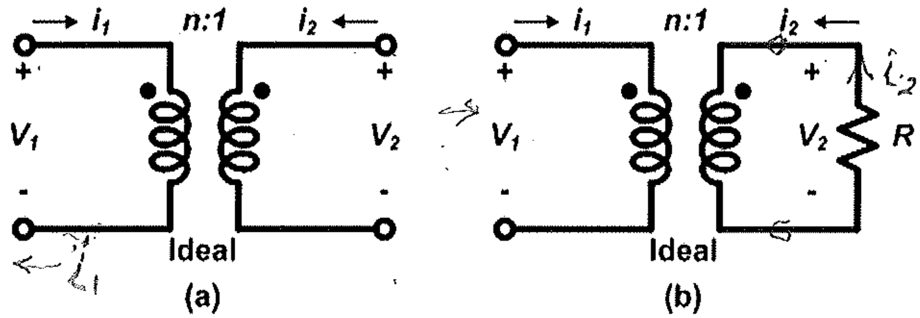
"Homework 1"

$E_C = ?$ Stored energy

$E_V = ?$ Delivered energy

(B) **Ideal Transformer:**

Port
Two-port



$V_2 = -i_2 R_2$

Fig. 6 An ideal transformer

Defined in terms of the following v-i relationships:

turn ratio

$$v_1 = nv_2 \tag{70a}$$
$$i_2 = -ni_1 \tag{70b}$$

or

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \tag{70c}$$

$$v_1 = nv_2 = -nRi_2 = (n^2 R)i_1 \tag{71}$$

$R_{in} = V_1 / i_1 = n^2 R$

At the input terminals, then, the equivalent resistance is $n^2 R$. Observe that the total energy delivered to the ideal transformer from connections made at its terminals will be

$$E(t) = \int_{-\infty}^{t} \underbrace{(v_1(x)i_1(x) + v_2(x)i_2(x))}_{P = 0} dx = 0 \tag{72}$$

Lossless, memoryless!

The right-hand side results when the v-i relations of the ideal transformer are inserted in the middle. Thus, the device is passive; it transmits, but neither stores nor dissipates energy.

Memoryless!

(C) "**Physical Transformer**":

L_1: primary self-inductance

M: mutual inductance

Henry

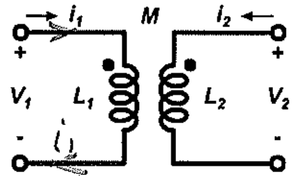


Fig. 7 A transformer

Two-port

The diagram is almost the same except that the diagram of the ideal transformer shows the turns ratio directly on it. The transformer is characterized by the following v-i relationships for the reference shown in Fig. 7:

mutual ind.

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \tag{73a}$$

primary self-ind.

And

$$v_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt} \tag{73b}$$

Thus it is characterized by three parameters: the two self-inductances L_1 and L_2, and the mutual inductance M. The total energy delivered to the transformer from external sources is

$$\begin{aligned} E(t) &= \int_{-\infty}^{t}[v_1(x)i_1(x) + v_2(x)i_2(x)]dx \\ &= \int_0^{i_1} L_1 i_1' di_1' + \int_0^{i_1,i_2} M d(i_1' i_2') + \int_0^{i_2} L_2 i_2' di_2' \\ &= \frac{1}{2}(L_1 i_1^2 + 2M i_1 i_2 + L_2 i_2^2) \ge 0 \end{aligned} \tag{74}$$

$x \triangleq i_1/i_2$

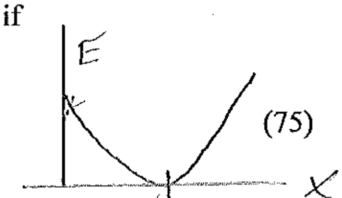
$L_1x^2 + 2Mx + L_2$

due to physical considerations!

It is easy to show that the last line will be non-negative if

"Homework 2"

$$\frac{M^2}{L_1 L_2} = k^2 \le 1 \tag{75}$$



Since physical considerations require the transformer to be passive, this condition must apply. The quantity k is called the *coefficient of coupling*. Its maximum value is unity for a closely-coupled transformer.

core

closely

A transformer for which the coupling coefficient takes on its maximum value $k = 1$ is called a *perfect*, or *perfectly coupled*, transformer. A perfect transformer is not the same thing as an ideal transformer. To find the difference, turn to the transformer equations (73) and insert the perfect-transformer condition $M = \sqrt{L_1 L_2}$; then take the ratio v_1/v_2. The result will be

$$\frac{v_1}{v_2} = \frac{L_1 \dfrac{di_1}{dt} + \sqrt{L_1 L_2}\,\dfrac{di_2}{dt}}{\sqrt{L_1 L_2}\,\dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt}} = \sqrt{L_1/L_2}. \tag{76}$$

This expression is identical with $v_1 = nv_2$ for the ideal transformer† if

$$n = \sqrt{L_1/L_2}. \tag{77}$$

Next let us consider the current ratio. Since (73) involve the derivatives of the currents, it will be necessary to integrate. The result of inserting the perfect-transformer condition $M = \sqrt{L_1 L_2}$ and the value $n = \sqrt{L_1/L_2}$, and integrating (73*a*) from 0 to t will yield, after rearranging,

$$i_1(t) = -\frac{1}{n}\, i_2(t) + \left\{\frac{1}{L_1}\int_0^t v_1(x)\,dx + \left[i_1(0) + \frac{1}{n}\, i_2(0)\right]\right\}. \tag{78}$$

This is to be compared with $i_1 = -i_2/n$ for the ideal transformer. The form of the expression in brackets suggests the *v-i* equation for an inductor. The diagram shown in Fig. 8 satisfies both (78) and (76). It shows how a perfect transformer is related to an ideal transformer. If, in a perfect transformer, L_1 and L_2 are permitted to approach infinity, but in such a way that their ratio remains constant, the result will be an ideal transformer.

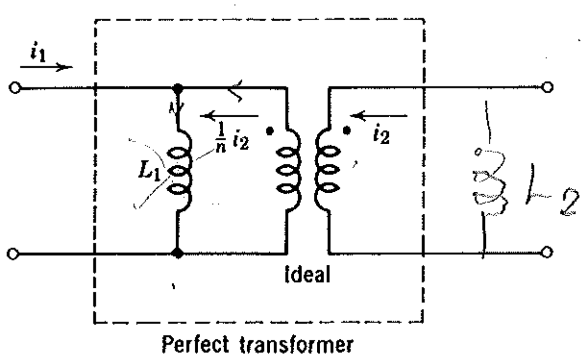


Fig. 8. Relationship between a perfect and an ideal transformer.

Losssless, memoried element.

(D) **The Gyrator:**

gyration res.

Definitions:

- **Port**: Two terminals, both input leads always carrying the same current.
- **Gyrator**: A two port network requiring active components for realization.

or R

Two-port

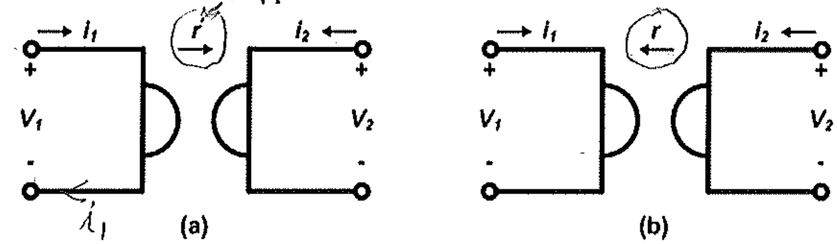


Fig. 9 A gyrator

$V(j\omega) = 1 + j2$

Often used to transform (convert) impedance into a different kind. Generally,

Time domain analysis

$v_1(t)$

$$Z_{in} = \frac{r^2}{Z_{load}}$$

$j\omega$ or s domain

For Fig. 9(a)

$$\begin{aligned} V_1 &= -ri_2 \\ V_2 &= ri_1 \end{aligned} \quad \text{or} \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{79a}$$

s or $j\omega$

$Z_{in} = \frac{r^2}{Z}$

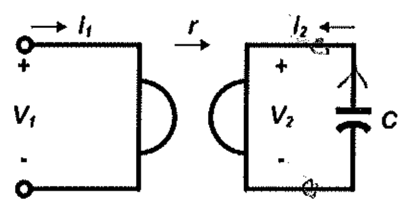
For Fig. 9(b)

$$\begin{aligned} V_1 &= ri_2 \\ V_2 &= -ri_1 \end{aligned} \quad \text{or} \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{79b}$$

$$E(t) = \int_{-\infty}^{t} (v_1 i_1 + v_2 i_2) dx = \int_{-\infty}^{t} [(-ri_2)i_1 + (ri_1)i_2] dx = 0 \tag{80}$$

Lossless
Memoryless
Non-reciprocal

L →



$i_2 = -C\frac{dv_2}{dt}$

Fig. 11 Gyrator terminated in a capacitor C

$i_2 = -C\frac{dv_2}{dt}$. Therefore, upon inserting the v-i relations associated with the gyrator, we observe that

$$v_1 = -ri_2 = -r\left(-C\frac{dv_2}{dt}\right) = rC\frac{d(ri_1)}{dt} = r^2C\frac{di_1}{dt} \triangleq L\frac{di_1}{dt} \tag{82}$$

$r = 1\,k\Omega$

$L \triangleq r^2C$

$i_1 = 1\,mA \to V_2 = 1V$

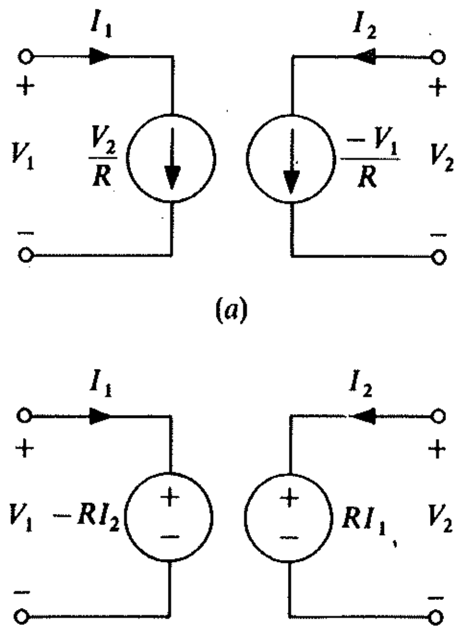
$i_2 = 1\,mA \to V_1 = -1V$

non-reciprocal!

Philips filters

(The first one is more practical, using transconductors)

$r = R$



VCCS
Transconductor
G_M

CCVS

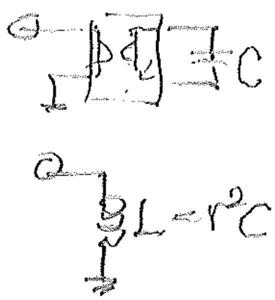
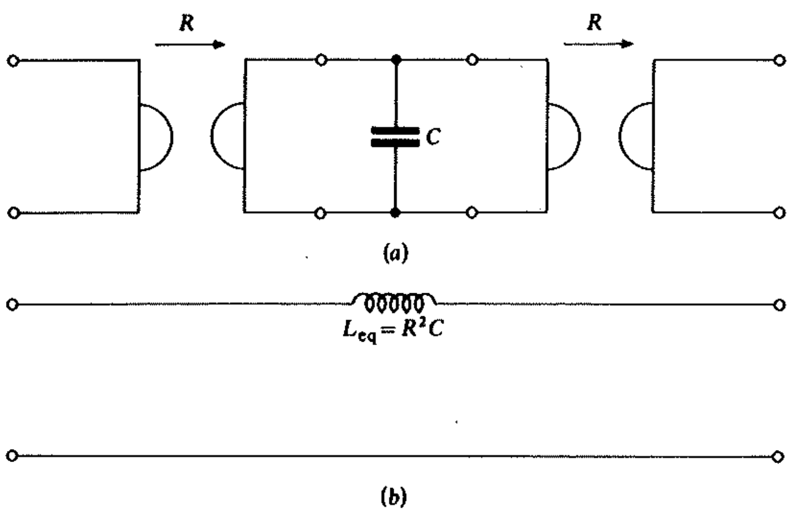


Figure 7-18 Ideal gyrator circuit



"Homework 3"

Figure 7-24 Floating-inductor simulation using gyrator

The Riordan circuit using two op-amps:

Laplace domain!

$V(s)$

$I(s)$ $\quad Z = \dfrac{V(s)}{I(s)}$

$s = j\omega \rightarrow$ phasor

$V = 3 \sin 2\omega t$

$i = \cos 2\omega t$

$Z \neq 3 \tan 2\omega t$

What is $Z(j\omega)$?

" Homework 4 "

$I_5 = \dfrac{V}{Z_5}$

GIC

opamps must be stable, ideal CMRR !?

Floating opamps

$V_{CM} = V$

Linearity!

gyrator

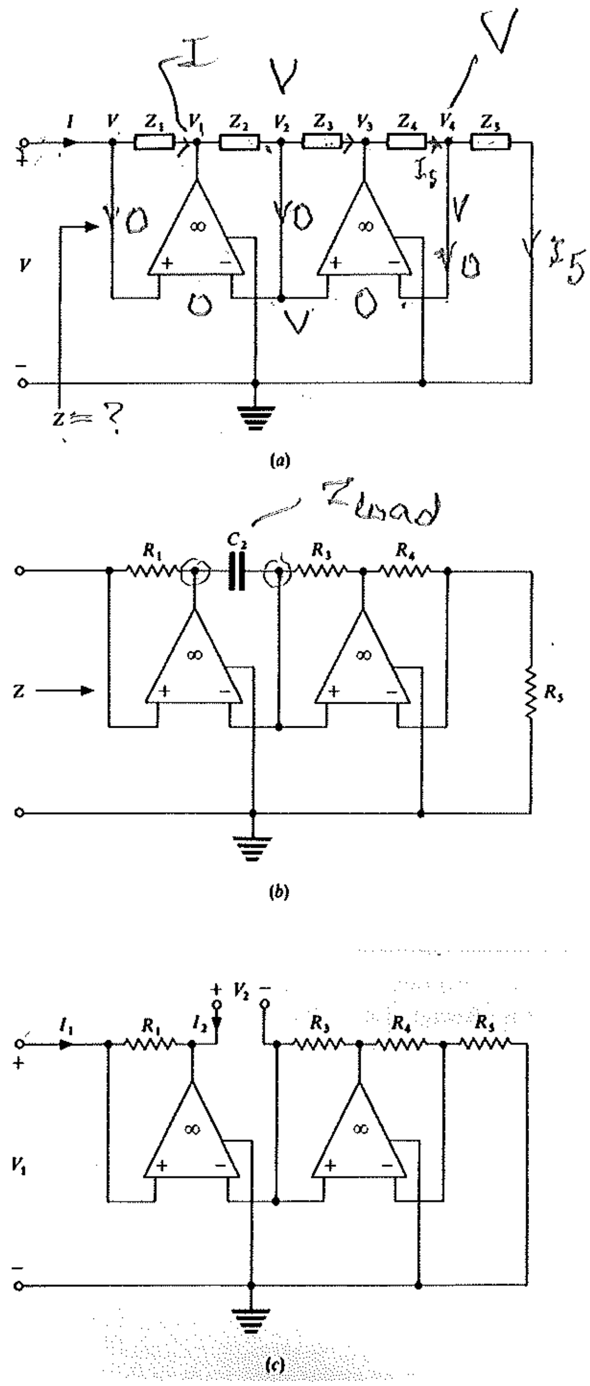


Figure 7-19 The Riordan circuit: (a) basic circuit;

(b) used as an inductor; (c) used as a gyrator

A circuit which uses two grounded-output op-amps and is useful for the realization of either GICs or GIIs is shown in Fig. 7-19*a*.† The input impedance Z can easily be found, as follows. When we recall that the input voltage of an op-amp is very nearly zero,

$$V \approx V_2 \approx V_4 \tag{7-62}$$

is obtained. Also, if we denote the current through Z_1 by I_1 (with the reference direction pointing left to right), the current through Z_2 by I_2, etc., clearly

$$\begin{aligned} I_1 &\approx I & V - V_1 &= I_1 Z_1 \approx V_2 - V_1 = -I_2 Z_2 \\ I_3 &\approx I_2 & V_2 - V_3 &= I_3 Z_3 \approx V_4 - V_3 = -I_4 Z_4 \\ I_5 &\approx I_4 & V &\approx V_4 = I_5 Z_5 \end{aligned} \tag{7-63}$$

Here we assumed, as usual, that the current in the input leads of the op-amps is zero.

Working backward in (7-63) leads to

$$V \approx I_5 Z_5 \approx I_4 Z_5 \approx -I_3 \frac{Z_3}{Z_4} Z_5 \approx -I_2 \frac{Z_3}{Z_4} Z_5 \approx I_1 \frac{Z_1 Z_3}{Z_2 Z_4} Z_5 \approx I \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \tag{7-64}$$

Hence

$$Z = \frac{V}{I} \approx \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \tag{7-65}$$

If Z_5 is regarded as a load impedance, the circuit behaves like a GIC; (7-46) takes the form

$$Z(s) = f(s) Z_5(s) \qquad f(s) \equiv \frac{Z_1(s) Z_3(s)}{Z_2(s) Z_4(s)} \tag{7-66}$$

If, for example, $Z_1 = R_1$, $Z_2 = 1/sC_2$, $Z_3 = R_3$, $Z_4 = R_4$, and $Z_5 = R_5$ (Fig. 7-19*b*), then $f(s) = R_1 R_3/[(1/sC_2)R_4]$ and

$$Z = \frac{R_1 R_3}{(1/sC_2)R_4} R_5 = s \frac{R_1 C_2 R_3 R_5}{R_4} \;\;\hat{=}\;\; sL \tag{7-67}$$

Hence, the input impedance is that of an *inductor*, with an equivalent inductance value $L_{eq} = R_1 C_2 R_3 R_5 / R_4$.

As (7-67) suggests, and as can be directly verified from (7-65), the two-port formed by regarding the terminals of Z_2 as an output port is a *gyrator* if all other impedances are purely resistive (Fig. 7-19*c*). More generally, if the terminals of Z_5 (or Z_1 or Z_3) constitute the output port, the circuit of Fig. 7-19*a* is a GIC; if the terminals of Z_2 (or Z_4) form the output port, the resulting two-port is a GII.

Assume now that we choose Z_2 and Z_4 as capacitive and Z_1, Z_3, and Z_5 as resistive impedances. Then (7-65) gives, for $s = j\omega$,

$$Z(j\omega) = \frac{R_1 R_3 R_5}{(1/j\omega C_2)(1/j\omega C_4)} = -\omega^2 R_1 C_2 R_3 C_4 R_5 \tag{7-68}$$

FDNR

We note that $Z(j\omega)$ is pure real, negative, and a function of ω. Such an impedance† is called a *frequency-dependent negative resistance* (FDNR). A slightly different form of FDNR can be obtained, e.g., by choosing Z_1 and Z_3 as capacitors and Z_2, Z_4, and Z_5 as resistors. Then

$$Z(j\omega) = -\frac{R_5}{C_1 R_2 C_3 R_4} \frac{1}{\omega^2} \tag{7-69}$$

As we shall see later, FDNRs are very useful for the design of active filters.